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ABSTRACT

The applicability of catastrophe theory to research in higher education is considered, with several problems that typically appear in the literature presented in a theoretical framework. A catastrophe model is attempted for each. The nature of mathematical modeling and the relationship between modeling continuous systems and discontinuous systems are assessed. Characteristics that must be present in a system for it to be suitable for a catastrophe theory based model are bimodality, divergence, discontinuity or catastrophe, hysteresis, and inaccessibility. Attention is also directed to interpreting a catastrophe surface. Four issues pertaining to the use of catastrophe theory as a tool for modeling behavior in the social sciences are as follows: its foundations of the theory in mathematics and natural philosophy: the assumptions needed for its application: details surrounding specific applications: and the attitudes, style, and intellectual honesty of its advocates. Hypothetical examples for applying the theory are considered: faculty vitality as a function of promotion difficulty and percent of junior faculty hired, teacher expectations as a function of student communication and performance, and student attrition as a function of previous academic performance and educational aspirations. Based on the literature on catastrophe theory, it is suggested that the bridges between the theoretical applications and those actual applications where theory can be tested through quantification have begun to be built. It is concluded that the usefulness of the theory may come from the ability of its models to provide more complete, and possibly more precise, description than appears possible with theories describing continuous behavior. Additionally, graphs based on the theory provide good conceptual tools. A bibliography is appended. (SW)

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CATASTROPHE THEORY IN HIGHER EDUCATION RESEARCH

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The purpose of this research is to consider the applicability of catastrophe theory to research in higher education. The review of the literature concentrates on those efforts in which the variables and their use seem to have an analogy to higher education. Several problems which typically appear in the literature are presented in a theoretical framework, and a catastrophe theory model is attempted for each. In this way, the researchers were able to consider the fundamental assumptions about a given model and its associated variables and assess, at a theoretical level, the appropriateness of applying catastrophe theory to the model.

CATASTROPHE THEORY IN HIGHER EDUCATION RESEARCH

Introduction

The purpose of this paper is to consider the applicability of catastrophe theory to research in higher education. The paper begins with a brief discussion on the nature of mathematical modeling and the relationship between modeling continuous systems and modeling discontinuous systems. The characteristics which must be present in a system for it to be suitable as a candidate for a catastrophe theory based model are presented, followed by some comments on interpreting a catastrophe surface. The controversy surrounding the use of catastrophe theory as a tool for research in the behavioral and social sciences is reviewed, and several problems which typically appear in the literature are presented in a theoretical framework with a catastrophe model proposed for each. Conclusions are drawn about the nature of catastrophe models and their applicability to research in higher education.

Modeling

Mathematical modeling has been associated with savings in terms of costs, risks, and time, and with increased understanding of the principles and relationships involved in the systems being modeled (Lawrence and Service, 1977). The use of such devices has ranged from those which are purely descriptive to those which are predictive, frequently designed in the later case to permit the researcher or user of a model to attempt to answer "what if" kinds of questions. By their very nature predictive models can be considered descriptive when the parameters reflect the reality of the present situation. The parameters can be conceptual or qualitative, as in most of the models in Collins (1976), or purely

quantitative as in those found in institutional models such as RRPM, SEARCH, and EFPM.

In most models today, a common underlying assumption is that the behavior being modeled is continuousand that a stable, predictable behavior can be found at any point on a continuum of possible behaviors. That is, the transition from one behavior to another is "smooth", with the behavior being modeled passing through all states between two given behaviors rather than suddenly "jumping" from one behavior to another.

Unfortunately the assumption of continuity fails all too frequently in actual practice. The literature points out that examples exist in the physical world where branches snap suddenly under pressure from the wind, beams buckle under too heavy a burden, and a cornered animal apparently wishing to flee will suddenly turn and attack when some sequence of events convinces the animal that flight is no longer the appropriate option. Expansion of the above examples, and others like them, can be found in Flay (1978), Panati (1976), and Zeeman (1976).

The observation that a behavior is discontinuous,or more generally that a behavior can occupy more than one state given seemingly identical control parameters, leads one who is attempting to build a model of the behavior away from the more traditional regression analysis based techniques. Recent efforts suggest, however, that one need not abandon the idea of building a model of a behavior simply because it has been discovered, observed, or suspected that the behavior contains one or more discontinuities (Fararo, 1978; Flay, 1978; Zeeman, 1977). These efforts are directed toward applying catastrophe theory to behavioral science, the mathematical foundations of which can be found in Thom (1972).

The Problem

The questions at hand are whether there are phenomena in higher educations which lend themselves to modeling through the use of techniques available in catastrophe theory and whether the theory is a useful alternative to existing theories. That is, are there behaviors which contain discontinuities or inherent instabilities and can we make the appropriate assumptions about these behaviors so that a catastrophe theory based model becomes a correct and useful tool for analysis?

Examples of Use

A search of the literature reveals a large number of applications, indicating a wide range of possibilities for catastrophe theory modeling. Examples exist in the biological sciences, the physical sciences, chemistry, mathematics, and the behavioral and social sciences (Thom, 1972; Woodcock and Davis, 1978). Among the social sciences have been the applications involving a prison riot (Zeeman, 1977), political party support (Woodcock and Davis, 1978), committee behavior (Zeeman, 1980), and decision making (Zeeman, 1980). Other examples include a study involving loyalty to a particular brand product based on the price of the item and its quality (Chidley, Lewis, and Walker, 1978), student evaluation of courses as a function of instructors' teaching ability and student commitment to the course (Frey and Flay, 1978), public support for a political candidate based upon social forces causing public opinion to polarize versus those causing one viewpoint to dominate (Cobb, 1978), and institutional vitality as a function of funding levels and demand (Johnson, 1980). Woodcock and Davis developed one model which demonstrated the effects of the anticipation of inflation and unemployment on the actual rate of inflation, another model which demonstrated the effects of the intensity of a stimulus versus

a sense of control over the stimulus on a subjects's response to the stimulus, and another which illustrated the impact of wealth versus family background on social status (Woodcock and Davis, 1978).

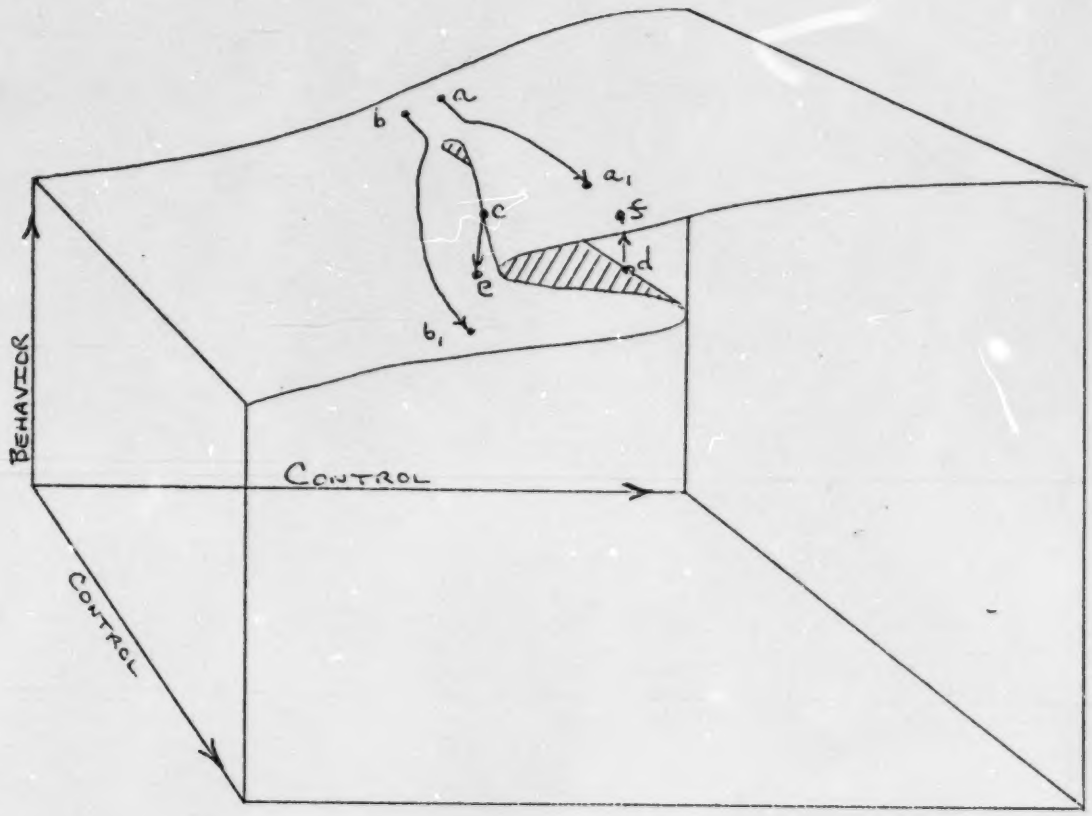
Although it has not yet been applied extensively to the field of education, possibly due to the difficulty in discerning control factors for particular behaviors, the apparent versatile applicability of the theory is indicative of its potential value in research in higher education. A key question is how to clearly identify those characteristics of a given system or situation which would suggest that catastrophe theory is, in fact, the appropriate analytical tool.

Characteristic Features of Catastrophe Models

A brief review of five properties which are common to cusp (and therefore higher order) catastrophes follows. More detailed descriptions can be found in Zeeman (1976, p.76; 1980, p.10, 13), Woodcock and Davis (1978, pp. 45-50), or for a more mathematical treatment, Fararo (1978,pp.307-311). Existence of any of the properties (bimodality, divergence, discontinuity or catastrophe, hysteresis, inaccessibility) suggests the possibility that one or more of the others may also exist.

Bimodality refers to the existence of two stable behaviors given a fixed subset of control variables. Graphically, bimodality means that for a particular pair of values on the control surface, behavior can be on either the upper sheet or the lower sheet, depending on the path leading to the behavior (see Figure 1, points a_1 and b_1).

Consideration of the path leading to a given behavior leads naturally to a discussion of the next two properties: divergence and discontinuity. Divergence refers to the property whereby two behaviors, beginning at almost identical points in the model, at a or b for example, and with seemingly identical (or almost identical) changes in the control variables,



Characteristic features of catastrophe models

Figure 1.

end up at very distant points within the model, at a_1 or b_1 for example. Both behaviors change continuously, and are in stable states for any given set of control variables. Discontinuity occurs when control variables cause the behaviors represented by either a_1 or b_1 to move past the behavior represented at an edge of a pleat (points c and d, for example). The discontinuity is that the behavioral change becomes abrupt, represented graphically by moving from c to e or from d to f.

Finally, hysteresis refers to a condition whereby the behavioral shift from the upper surface to the lower surface does not occur at the same point as the shift from the lower surface to the upper surface. Inaccessibility refers to those behavior values which are unstable. That is, the system cannot come to rest at a behavioral point defined in the inaccessible zone, hence the discontinuity. Inaccessible behaviors are those which are "jumped" by the catastrophe, and are represented graphically as the set of behaviors on the inner fold of the pleat.

A hypothesis that the above properties are contained in a behavorial system can apparently be made only through inspection and reflection. Given the hypothesis, and some definition of the behavorial and control variables, Cobb (1981), provides a method for determining whether a cusp exists and for an estimation of the parameters of a canonical form of the cusp. A number of questions remain however, perhaps the most significant of which is the actual interpretation of a behavorial surface.

On the Interpretation of a Surface

Thom's theory is based on the concept of structural stability, and the resulting social science models which have been built by Zeeman and others are qualitative in nature. As such, they generally have no scale and are conceptual abstractions.

Thus given the present state of the art, a catastrophe graph can be interpreted as a canonical representation of a behavior, and quantified relationships among various aspects of the behavior cannot be inferred from the apparent position of these aspects on a given surface. In fact, since some surfaces are topologically equivalent to others, it would seem that an interpretation based on the relative positions of the points describing the aspects of a behavior might change if one were to consider a different physical representation (i.e. equivalent topological surface) depicting the behavior. Furthermore, since most catastrophe models have no scale, we do not know if a given discontinuity is a predominant or minor, perhaps insignificant aspect of a behavior being modeled. That is, is the discontinuity an insignificant ripple on an otherwise smooth and continuous surface, thus becoming an uninteresting aspect of the behavior, or is there a significant aspect of the behavior which can best be described by one of the catastrophes? If "live" data were available, how many of the points on the control surface would actually be under the pleat?

The Controversy

Objections to the use of catastrophe theory as a tool for modeling behavior in the social sciences focus not on whether the theory contains enough rigor to permit its application, but on the methods in which researchers actually apply the theory in a given situation. Woodcock and Davis (1978), proponents of the theory, are more complete in their analysis of the argument than most. They suggest that there are four issues surrounding the theory: its foundations in mathematics and natural philosophy; the assumptions needed for its application; details surrounding specific applications; and the attitudes, style, and intellectual honesty of its advocates (Woodcock and Davis, 1978, p.62).

Details of the arguments will not be presented in this paper. In general opponents suggest that better models exist in those cases where catastrophe theory is used; that the assumptions of the existence of a potential and that its behavior depend on a limited number of control factors are unrealistic; and that the "uninformed" who would use the theory either apply it incorrectly, turn the model or the theory to suit their own ends, or simply appeal to a not well understood mathematical theory as a justification for conclusions which either do not necessarily follow or, in the worst case, are simply incorrect (Sussmann and Zahler, 1978, pp. 387-389). Fararo, a proponent of the theory, includes a less heated analysis of the problems involved with applying catastrophe theory to social science as part of an expository paper on the theory. Two key questions suggested by Sussmann and Zahler are reinforced by Fararo. The first relates to the problems of using ordinal-level data in dynamic system models (Fararo, 1978, p.312) and the second relates to the empirical identification of predicted catastrophe events (Fararo, 1978, p.315).

In addition to arguments and support of one particular application or another, proponents note that catastrophe theory is relatively new, that methods for using the theory are not well known, and finally that the process of modeling is in itself one of trial and error, working and reworking a particular model until it fits a particular situation (Woodcock and Davis, 1978).

That the basic structures provided by catastrophe theory are qualitative is clear. These structures, when appropriately linked to the information that we already know about a given system or situation, will almost surely help us know more about the dynamics of the system. For catastrophe models to become accepted, however, and for these models to become useful analytical tools, empirical tests involving hard data producing results that can be

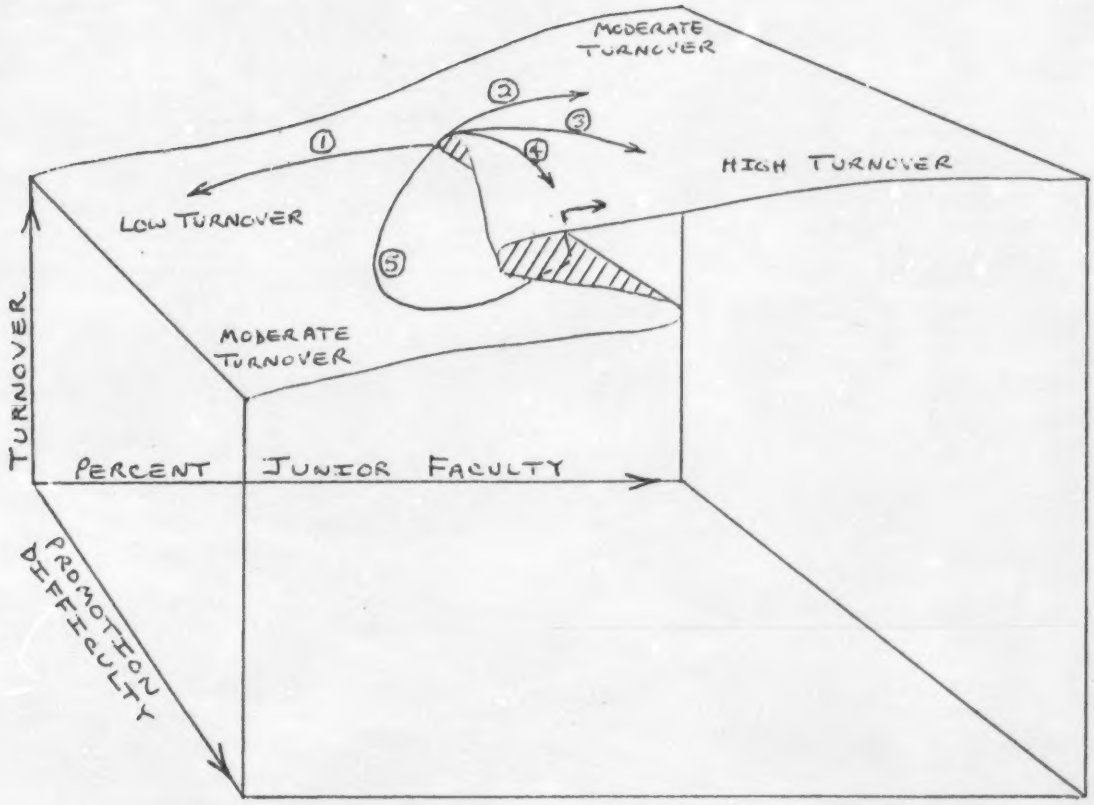
experimentally reproduced over time is required. Cobb, (1980, 1981) provides us with just such a statistical procedure. The process is not computationally complex and appears to go a long way toward responding to the objections of Sussman and Zahler.

Hypothetical Examples

Three examples follow. In each case a hypothetical discontinuity is introduced into previously published results.

Faculty Vitality Bruss and Kutina (1981, p.19-30), discuss faculty vitality in a period of retrenchment, suggesting strategies involving promotion, hiring, tenure, and contracts to achieve mandated expense reductions in budgets. The research suggests that promotion difficulty and the percent of junior faculty hired have a discernable effect on the turnover rate of the institution's extant faculty. Moreover, it appears that as the promotion difficulty and number of junior faculty members increase there is a possibility for a dangerously high level of faculty turnover.

A cusp model of this situation might take on the following characteristics (See Figure 2). Path 1 exhibits what might be the base case of a low percentage of junior faculty and a relatively liberal promotion policy. Turnover is low. Path 2 represents the case of an increased percentage of junior faculty and a more restrictive promotion policy. Turnover is somewhat higher. Path 3 follows the case in which there are a minimal number of senior faculty at the institution and promotion is relatively difficult, and Path 4 represents the case in which promotion has been made quite difficult and almost all replacements are filled by junior faculty. The discontinuity may occur if we hypothesize that at some point the level of promotion difficulty becomes so overpowering that even the senior faculty are faced with almost an impossibility of promotion. The lack of incentive may cause a catastrophic loss of senior faculty. Path 5 illustrates the result, a "jump" from a moderate to a high turnover rate.



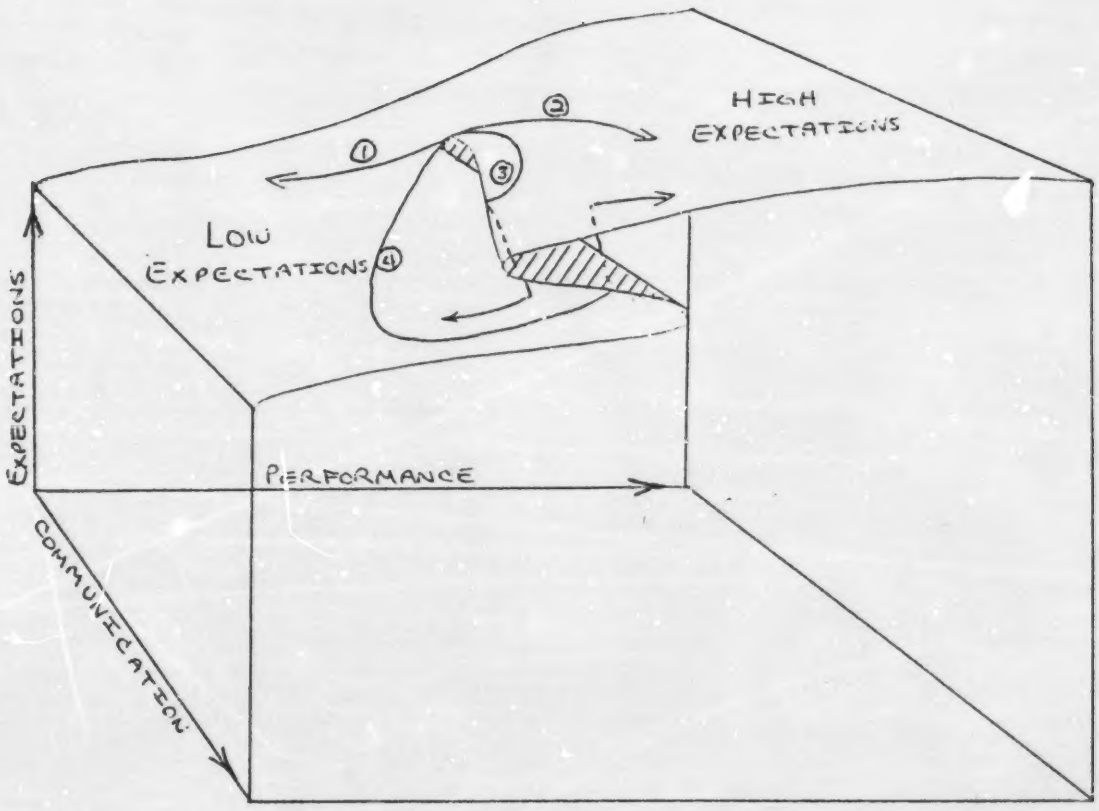
Faculty vitality as a function of promotion difficulty and percent of junior faculty hired

Figure 2.

Teacher Expectations and Student Performance An extensive review of the literature suggests that student communication and performance influence the expectations that a teacher has for his or her students (Cooper, 1979). That is, variations in perceived student ability and background lead teachers to expect different levels of performance. Reinforcement of these expectations (students who respond to questions in a favorable manner and who do well academically seem to receive more positive responses from the teacher than those who respond and perform less favorably) may influence student performance, thus generating an ongoing expectation/performance relationship. Cooper presents a model to describe the phenomenon and suggests that the development of other sequential models is appropriate.

Figure 3 represents a hypothetical catastrophe model with student performance and communication as the control variables and teacher expectations as the behavior variable. Several paths might exist in such a model. Path 1, for example, represents the case where performance and communication decrease correspondingly with the teacher's expectations of a student. On the other hand, where students have high performance levels and their in-class responses are deemed favorable the expectations are correspondingly high, as in Path 2.

It appears that student responses are inextricably linked in a circular fashion to the type of response that the student receives from the teacher. If the teacher smiles and allows an adequate response time for the student to answer then the teacher is likely to get a "better" answer. Similarly, the teacher who lacks warmth in conversing with a student and does not provide encouragement is likely to receive a poor response from the student. Such behavior may suggest that discontinuities exist in the model. In Path 3 for example, if a student, even a high performer, is perceived as not being bright and is treated as a slower student by a cool response from the



Teacher expectations as a function of student communication and performance

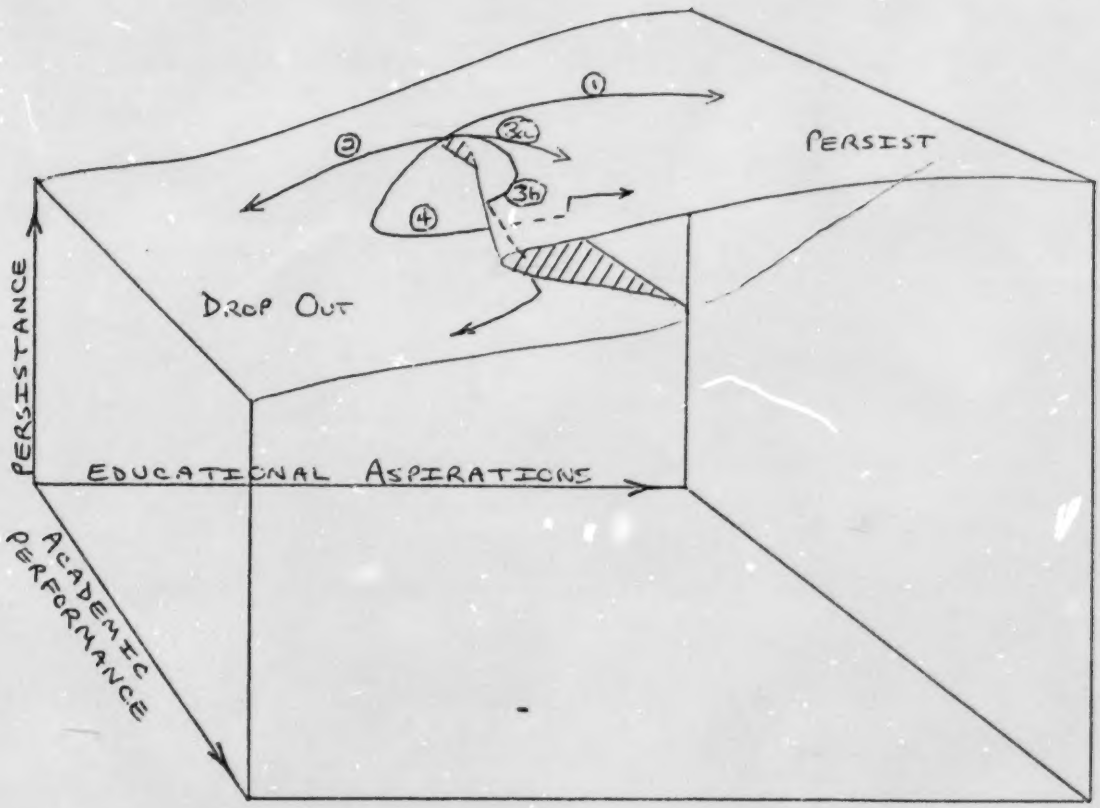
Figure 3.

instructor, then his communication level may drop, and hence the instructor's expectations. Of at least as great importance is noting what the model suggests may be the impact of a student receiving low expectations who is treated as if he were a brighter student. Path 4 indicates that the teacher's encouragement might tend to stimulate the student's communication so that performance improves. With improved performance and strengthened communication ties the student may then be prone to switch from a person of low expectations to one meriting high expectations.

Student Attrition According to Astin's study of college freshmen, the major indicators of a student's persistance in college are previous academic performance and educational aspirations (Astin, 1975, pp. 22-46). A cusp model of student persistance based on these two factors might take on the following characteristics (See Figure 4).

Since academic achievement and educational aspirations are positively related to persistance, those freshmen who are successful students and are committed to college are likely to remain associated with the institution. The situation is illustrated by Path 1. On the other hand, Path 2 represents a student with poor academic performance and low educational aspirations. The literature suggests that the student will drop out of school in a relatively short period of time.

Case 3 represents the potential discontinuity. It may be that a student performs well yet lacks interest in continuing in college (Astin notes other factors influencing this interest.). Forced to enroll, due to parental or peer pressure for example, the student becomes involved in an academic situation which is undesirable. As dissatisfaction builds the student turns from Path 3a to 3b, eventually becoming "turned off" and dropping out.



Student attrition as a function of previous academic performance and educational aspirations

Figure 4.

Path 4 illustrates a situation in which a student has already dropped out due to a poor academic background and relatively low aspirations. As the student's aspirations gradually increase, leading to a desire for a change of lifestyle, the student then returns with higher expectations, as shown in Path 4.

Some Comments on the Literature

Although catastrophe theory is a relatively new technique, the body of literature about the topic has grown to a point where a discretionary search of the literature is not only feasible but warranted. Zeeman (1976), provides an excellent starting point in an article which has become a classic exposition on the topic. The discussion is an example of the qualitative nature of the theory with the examples serving as canonical representations of the systems that they propose to model. Little mathematical sophistication is required of the reader. As a follow-up, the reader might wish to consider the address to the twentieth annual forum of the Association for Institutional Research (Zeeman, 1980). Woodcock and Davis (1978) provides probably one of the best books on the topic. The nature of the book is similar to Zeeman's work--no sophisticated mathematics is assumed throughout. In addition to an explanation of the qualitative nature of catastrophe theory, the elementary catastrophes, and the controversy surrounding the theory, a varied and detailed set of examples in both the physical and social sciences is included. Flay (1978), has added some good examples of theoretical applications to attitudes and social behavior as part of a paper which, as a supplement to the previous two, should provide the reader with a thorough understanding of the potentials, the problems, and the pitfalls involved in applying the theory through the social sciences.

The bridges between the theoretical applications and those actual applications where the theory can be tested through quantification have begun to be built. Guastello's (1981) paper is closer to theoretical constructs than

to actual applications. The work is important because it effectively deals with some of the objections to the models proposed by Zeeman. Fararo (1978), provides a relatively sophisticated mathematical treatment of the topic. This work is important because it begins to provide a bridge between the theoretical uses of the catastrophe models and actual physical applications which can be quantified, reproduced, and tested for validity. One does not have to understand all of the mathematics involved in order to comprehend the essence of the article. Finally, Cobb (1981), provides us with actual techniques for numerical applications of the theory.

The researcher contemplating a use for catastrophe theory should read carefully the critique provided by Sussman and Zahler (1978). All mathematical techniques can be incorrectly applied, and catastrophe theory is no different in that regard. Sussman and Zahler's work provides us with an excellent review of the problems to be avoided--although to follow their recommendations to a logical conclusion, one would not apply the theory at all.

Conclusion The usefulness of the theory may come from the ability of its models to provide more complete, and possibly more precise, descriptions than appear possible with theories describing continuous behavior. If nothing else, the graphs provide good conceptual tools. The process of modeling is, after all, one of trial and error in an effort to obtain the best mathematical description of a given situation. The test of the model comes only when the parameters change and the researcher has the opportunity to determine whether the model still fits, or whether it has broken down completely--no longer representative of reality.

The objection that models based on the theory require a potential and depend on a limited number of variables is true for all models. Catastrophe theory provides the ability to consider a richer and more complex relationship among the variables. Given the complexities of human behavior, it would appear

that a theory capable of accepting complex transformations on a great number of variables stands a better chance of reflecting reality than those theories which presume continuity and which deal with the relationships among a small number of variables.

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